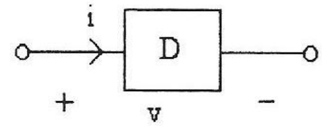
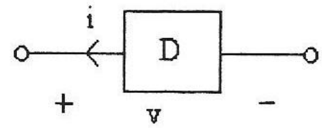
Power & the Passive Sign Convention [PSC]

Power into a device [or absorbed by the device]: $p_{in} = vi$
[using the passive sign convention]



Power to a resistor: $p_R = v\,i = i^2R = \dfrac{v^2}{R}$

Power out of a device [or delivered by the device]: $p_{out} = vi$
[not using the passive sign convention]



Kirchhoff's Laws

KVL: algebraic sum of voltages around a loop equ $p = \dfrac{dw}{dt}$: sum of all voltage drops around a loop equals zero.

KCL: algebraic sum of currents at a node equals zero; or sum of all currents leaving a node equals zero.

Series & Parallel Combinations

Voltage sources: in series: add the voltage waveforms
in parallel: the voltages must be the same

Current sources: in parallel: add the current waveforms
in series: the currents must be the same

Resistance in Parallel

$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}$$

ADD resistances in SERIES

$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k$$

ADD conductances in PARALLEL

$$G_{eq} = \sum_{i=1} G_i = G_1 + G_2 + \cdots + G_k$$

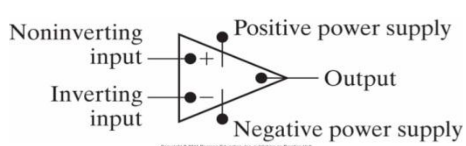
k

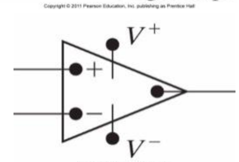
Two resistors in parallel:

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

TABLE 1.3 Standardized Prefixes to Signify Powers of 10		
Prefix	**Symbol**	**Power**
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

$$v = \frac{dw}{dq}; \qquad i = \frac{dq}{dt}$$





Ideal Op Amps

$V^- \leq v_o \leq V^+$

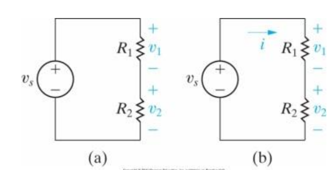
$i_p = i_n = 0$

$v_p = v_n$

Voltage Divider

$$v_1 = iR_1 = \left(\frac{R_1}{R_1 + R_2}\right) v_s$$

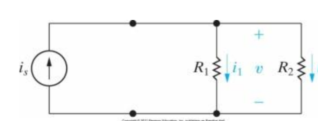
$$v_2 = iR_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$



Current Divider

$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i_s$$

$$i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$



MORE USEFUL FACTS

Method of Node Voltages:

1) ID nodes & select a reference node
2) define node voltages
3) write the VS equations*, using a supernode to handle a floating voltage source
4) write the KCL equations*
5) solve the equations in steps 3) & 4)

*To handle a dependent source, write the controlling signal in terms of the node voltages.

Method of Mesh Currents:

1) ID meshes
2) define mesh currents
3) write the CS equations*, using a supermesh to handle a shared current source
4) write the KVL equations*
5) solve the equations in steps 3) & 4)

*To handle a dependent source, write the controlling signal in terms of the mesh currents.

Thevenin equivalent: V_{TH} in series with R_{TH} — $V_{TH} = V_{OC}$ is the open circuit voltage.

Norton equivalent: I_N in parallel with R_N — $I_N = I_{SC}$ is the short circuit current.

Equivalence relations: $R_{TH} = R_N$ & $V_{TH} = R_{TH} I_N$ — $R_{TH} = R_N$ is the equivalent resistance.

Source transformation: replace a Thevenin equivalent with a Norton equivalent, or vice-versa

Maximum Power Transfer:

A load resistance R_L draws maximum power from a circuit when $R_L = R_{Th}$, where R_{Th} is the Thevenin resistance of the circuit to which the load resistance is connected.

TABLE 6.1 Terminal Equations for Ideal Inductors and Capacitors

Inductors	
$v = L\frac{di}{dt}$	(V)
$i = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$	(A)
$p = vi = Li\frac{di}{dt}$	(W)
$w = \frac{1}{2}Li^2$	(J)
Capacitors	
$v = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$	(V)
$i = C\frac{dv}{dt}$	(A)
$p = vi = Cv\frac{dv}{dt}$	(W)
$w = \frac{1}{2}Cv^2$	(J)

TABLE 6.2 Equations for Series- and Parallel-Connected Inductors and Capacitors

Series-Connected
$L_{eq} = L_1 + L_2 + \cdots + L_n$
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$
Parallel-Connected
$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$
$C_{eq} = C_1 + C_2 + \cdots + C_n$

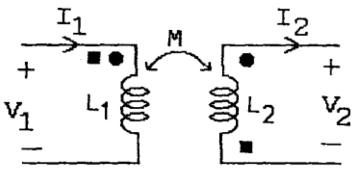
Parallel Inductors Initial Current

$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0) + \cdots + i_n(t_0)$$

Series Capacitors Initial Voltage

$$v(t_0) = v_1(t_0) + v_2(t_0) + v_3(t_0) + \cdots + v_n(t_0)$$

Mutual Inductance and Linear Transformers



Round dots:

$$\vec{V}_1 = j\omega L_1\vec{I}_1 - j\omega M\vec{I}_2$$
$$\vec{V}_2 = +j\omega M\vec{I}_1 - j\omega L_2\vec{I}_2$$

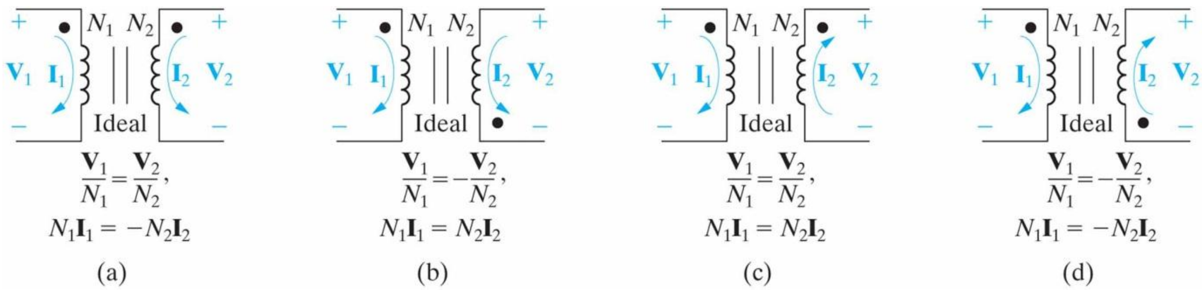
Square dots:

$$\vec{V}_1 = j\omega L_1\vec{I}_1 + j\omega M\vec{I}_2$$
$$\vec{V}_2 = -j\omega M\vec{I}_1 - j\omega L_2\vec{I}_2$$

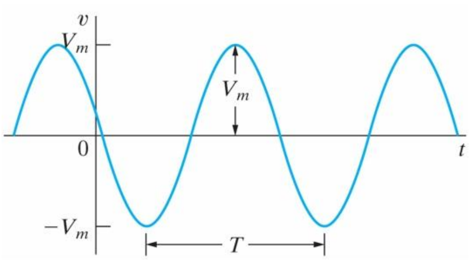
Reflected impediance $Z_r = \frac{\omega^2 M^2}{|Z_{22}|^2}[(R_2 + R_L) - j(\omega L_2 + X_L)]$ where $Z_{22} = R_2 + R_L + j(\omega L_2 + X_L)$

$$\boldsymbol{M = k\sqrt{L_1 L_2}} \qquad \boldsymbol{w(t) = \frac{L_1 i_1^2}{2} + \frac{L_2 i_2^2}{2} \pm i_1 i_2 M}$$

Ideal Transformers



The impedance seen by the source $Z_{in} = \frac{\boldsymbol{V}_1}{\boldsymbol{I}_1} = \frac{1}{a^2}\frac{\boldsymbol{V}_2}{\boldsymbol{I}_2} = \frac{1}{a^2}Z_L$ where $a = \frac{N_2}{N_1}$



$$v = V_m\cos(\omega t + \phi)$$
$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$
$$\boldsymbol{V = ZI}$$
$$\overrightarrow{V_{eff}} = Z\overrightarrow{I_{eff}}$$

TABLE 9.1 Impedance and Reactance Values

Circuit Element	Impedance	Reactance
Resistor	R	—
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$

Rectangular to Polar conversion

$$\boldsymbol{V} = A + jB = \sqrt{A^2 + B^2}\angle\left(\tan^{-1}\frac{B}{A}\right)^\circ$$

Polar to Rectangular conversion

$\boldsymbol{V} = V_m\angle\phi^\circ = V_m\cos\phi + jV_m\sin\phi$ (Note that V_m is a magnitude and should be positive, thus angle differs)

Complex Arithmetic

Addition/Subtraction – must be in rectangular coordinates; combine real terms and imaginary terms.
Ex.$(A_1 + jB_1) + (A_2 + jB_2) = (A_1 + A_2) + j(B_1 + B_2)$

<u>Multiplication</u> – Rectangular coordinates; distribute (FOIL)
Polar coordinates; multiply the magnitudes and add the angles

<u>Division</u> – Rectangular coordinates; multiply the denominator and numerator by the complex conjugate of the denominator, then simplify.

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Polar coordinates; divide the magnitudes and subtract the angles

Complex Number Identities $j^2 = -1$ and $\frac{1}{j} = -j$

Average Power

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = V_{eff} I_{eff}\cos(\theta_v - \theta_i) = \left|\overrightarrow{I_{eff}}\right|^2 R = \frac{1}{2} I_m^2 R = \frac{\left|\overrightarrow{V_{eff}}\right|^2}{R}$$

Reactive Power

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = V_{eff} I_{eff}\sin(\theta_v - \theta_i) = \left|\overrightarrow{I_{eff}}\right|^2 X = \frac{1}{2} I_m^2 X = \frac{\left|\overrightarrow{V_{eff}}\right|^2}{X}$$

Complex power $S = P + jQ$

$$S = \frac{1}{2} V_m I_m \angle(\theta_v - \theta_i) = V_{eff} I_{eff} \angle(\theta_v - \theta_i) = \overrightarrow{V_{eff}}\overrightarrow{I_{eff}}^* = \frac{1}{2}\vec{V}\vec{I}^* = \left|\overrightarrow{I_{eff}}\right|^2 Z$$

Apparent Power $|S| = \sqrt{P^2 + Q^2}$ Power factor $\mathrm{pf} = \cos(\theta_v - \theta_i)$

Lagging power factor: implies that the current lags the voltage – hence *inductive load*

Leading power factor: implies that the current leads the voltage – hence *capacitive load*